

# Graduate Public Finance

## Capital Taxes

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Princeton

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### Lecture 7

Thanks to Fullerton and Ta, David Albouy, Alan Auerbach, Raj Chetty, Kevin Murphy, Emmanuel Saez, Gabriel Zucman, and Treb Allen for providing their notes, some of which are reproduced here. Stephanie Kestelman provided excellent assistance making these slides.

## 1 Brief Introduction

- What is capital income?
- The level and distribution of capital income
- How is capital income taxed in the US?

## 2 Capital Tax Incidence: simplest possible toy models

- Supply and demand in the capital market
- Brief aside on rental and asset markets for capital
- Simple spatial model: One factor, two locations

## 3 Capital Tax Incidence: Harberger

- Fullerton and Ta (2017)
  - Consumers and Producers
  - Equilibrium
  - Welfare loss from taxation
  - Understanding equilibrium (graphical and quantitative analysis)
  - Effect of Tax on Corporate Output
  - Effect of Tax on Capital
  - Effect of Tax on Corporate Capital
- Harberger Model (more general utility and technology)

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Equity-efficiency tradeoffs of capital taxation seem especially stark

## 1 Efficiency

- Capital taxes reduce scale of economic activity
- Capital accumulation, which may be highly responsive to rates of return, is correlated with economic growth
- Capital mobility: taxes can lead to misallocation

## 2 Equity

- Distribution of capital income is much more unequal than labor income
- Capital mobility: burden may be shifted to labor

## 3 Evidence

- Empirical evidence/our understanding of capital taxation is less well developed than labor income taxation

## 4 Policy Relevance

- Future of fiscal policy (taxing robots, driverless cars, corp tax reform)
- Destination-based cash flow taxes, international reforms

# What is capital income?

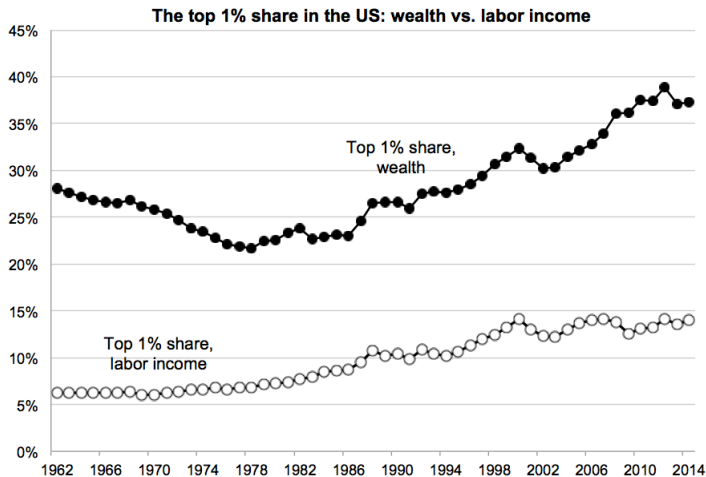
Individuals derive market income (before tax) from **labor** and **capital**:  
 $z = wl + rk$  where  $w$  is wage,  $l$  is labor supply,  $k$  is wealth,  $r$  is rate of return on wealth

- 1 **Labor income inequality** is due to differences in working abilities (education, talent, physical ability, etc.), work effort (hours of work, effort on the job, etc.), and luck (labor effort might succeed or not)
- 2 **Capital income inequality** is due to differences in wealth  $k$  (due to past saving behavior and inheritances received), and in rates of return  $r$  (varies dramatically over time and across assets)

## Level and distribution of capital income (1/2)

- Labor income  $wl \simeq 75\%$  of national income  $z$
- Capital income risk  $rk \simeq 25\%$  of national income  $z$  (has increased in recent decades)
- Wealth stock  $k \simeq 400\% - 500\%$  of national income  $z$  (is increasing)
- Rate of return on capital  $r \simeq 5\%$
- $\alpha = \beta \cdot r$  where  $\alpha = rk/z$  share of capital income and  $\beta = k/z$  wealth to income ratio
- In GDP, gross capital share is higher (35%) because it includes depreciation of capital ( $\simeq 10\%$  of GDP)
- National Income = GDP - depreciation of capital + net foreign income

# Level and distribution of capital income (2/2)



Source: Piketty, Saez and Zucman (2016).

# Composition of wealth

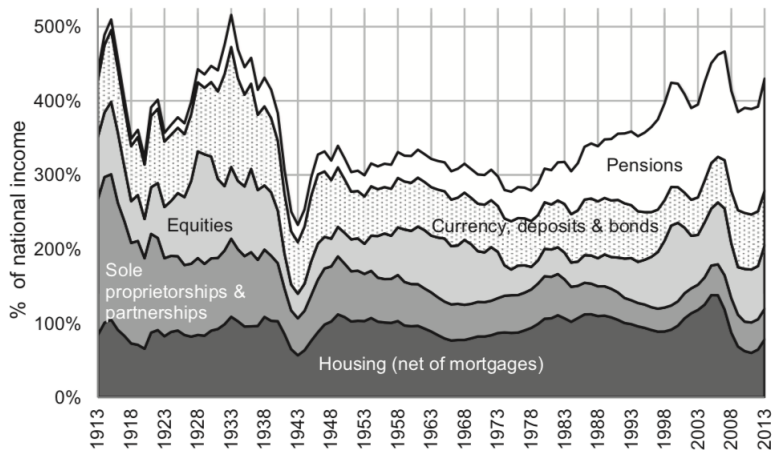


FIGURE II

Aggregate US Household Wealth, 1913–2013

Source: Saez Zucman (2016)



# Composition of capital income

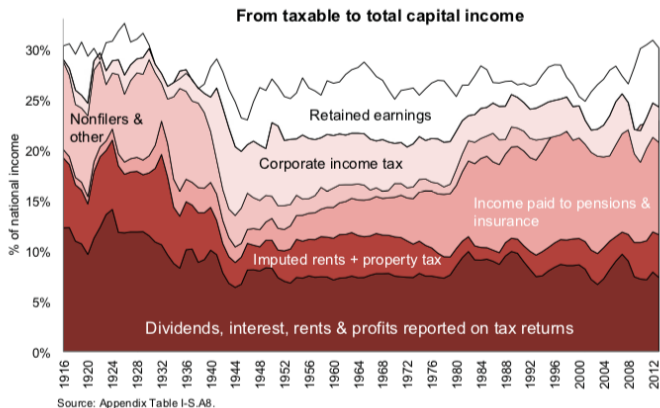


FIGURE I

From Taxable Income to National Income (1916–2014)

Source: Piketty Saez Zucman (2018)

# Where do these numbers come from?

E.g.: PSZ's retained earnings imputation (see Smith Yagan Zidar Zwick, 2018 for details)

PSZ method to allocate \$2.1T of flow of equity income

- 1 Compute share accruing to top 1% ( $= \frac{\$8.6T}{\$26.2T} = 33\%$ )
  - Top S+C wealth/equity wealth in hh, non-profit, pensions
  - Top S wealth of \$1.5T ( $= \frac{\$192B}{\$355B} \times \$2.8T$ )
  - Top C wealth of \$7.1T ( $= \frac{\$114+490B}{\$255+687B} \times \$11T$ )
- 2 Divide top 1 flow of \$710B into proportional contributions
  - Top retained earnings of \$197B ( $= \frac{\$7.1T}{\$23.4T} \times \$649B$  of RE)
  - Similar steps  $\rightarrow$  top C-divs of \$169B,
  - Similar steps  $\rightarrow$  top S-divs of \$224B.
  - Top RE share is 33% ( $= \frac{\$197B}{\$197+169+224B}$ )

Thus, top 1 retained earnings of **\$235B** ( $= \$2.1T \times .33 \times .33$ )

# How much is it taxed?

In the US, total capital taxes can be decomposed into three categories of roughly equal importance:

- 1 Corporate tax = 3% of  $Y$  (around 20% of a 15% tax base)
- 2 Annual property rates = 3% of  $Y$  (around 1% of a 300% tax base)
- 3 Personal taxes on a capital income = 2.8% of  $Y$  (around 30% of a  $15\% \times 60\% = 9\%$  tax base) + estates = 0.2% of  $Y$  (around 2% of a 10% tax base)

Won't be able cover all of these in as much depth as I'd like, especially on optimal capital taxation and classic results like Chamley Judd [or Straub Werning (2018)'s reassessment]. We'll mostly focus on 1 and Henrik's lecture on 11/27 will discuss 3.

I strongly encourage you to review Stancheva's capital tax lecture slides:  
[http://scholar.harvard.edu/files/stancheva/files/capital\\_taxation.pdf](http://scholar.harvard.edu/files/stancheva/files/capital_taxation.pdf)

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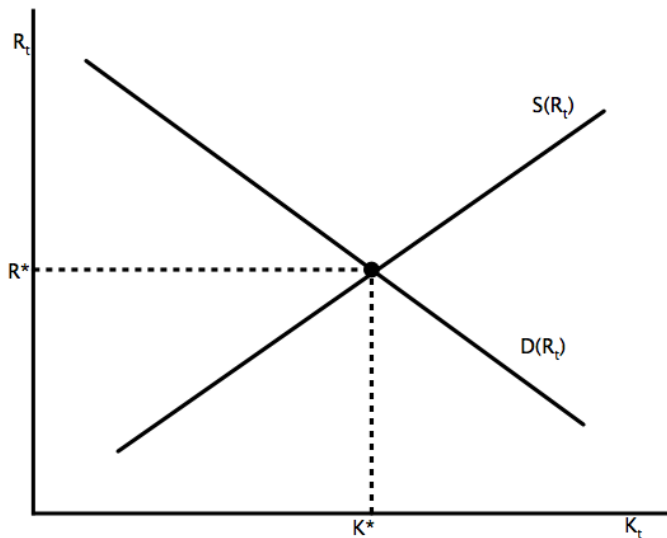
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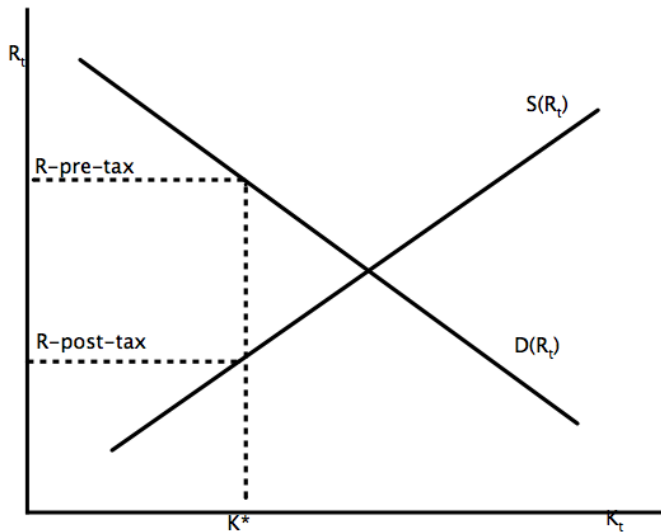
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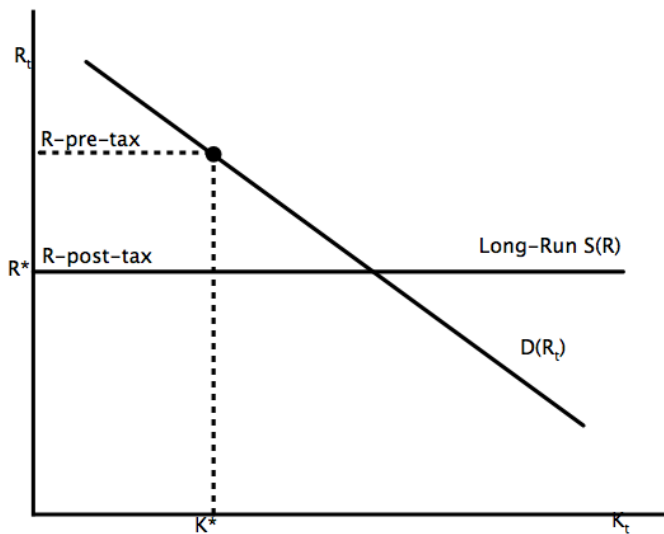
# Impact of a Capital Tax



# Impact of a Capital Tax



# Impact of a Capital Tax (in Long Run)



Who bears the capital tax in the long run?

- Who gets the triangle above R-pre-tax (i.e., consumer surplus in the typical S and D graph)?
- If firms don't earn profits, this all goes to workers in terms of higher wages or lower prices
- A key object is the **elasticity of capital supply**, is likely larger (and some think infinite) in the LR
- Note that the distortion in the capital market reduces surplus more than it increases tax revenues (as with most taxes)
- Finally, distortions due to capital taxation are often considered in a dynamic context in which the distortion compounds overtime (See Ivan Werning's recent paper on the classic Chamley-Judd results)



# Aside on capital markets

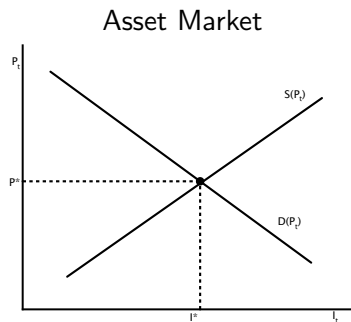
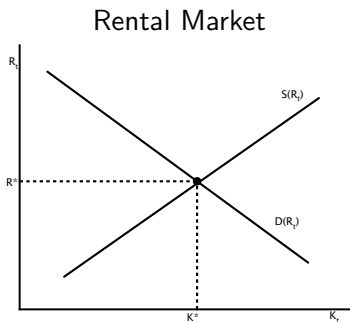
# Capital markets: 4 key equations

We will use 4 equations to analyze capital markets

- 1 **Stock Adjustment:** the amount of capital today depends on how much there was yesterday, depreciation, and new investment
- 2 **Asset pricing equilibrium:**<sup>1</sup> the rental price of using an asset is simply the cost of buying the good and re-selling it after one period
- 3 **Rental market equilibrium:** the demand for using capital services is downward sloping
- 4 **Investment market equilibrium:** the supply of capital assets is upward sloping

# Rental and asset markets are linked

Use the link between rental and asset markets to analyze capital markets



where  $R_t$  is the **rental price** of using capital services  $K_t$  and  $P_t$  is the **purchase price**, which depends on the level of investment  $I_t$ .

## 4 key equations

- 1 **Stock Adjustment:**  $K_t = (1 - \delta)K_{t-1} + I_t$
- 2 **Asset pricing equilibrium** The rental cost of using an asset is simply the cost of buying the good and re-selling it after one period
- 3 **Rental market equilibrium:**  $K = D(R)$
- 4 **Investment market equilibrium:**  $I = S(P)$

## 2. Asset pricing equilibrium (without taxes)

### What is the relationship between rental and capital prices?

The rental cost of using an asset is simply the cost of buying the good and re-selling it after one period

$$R_t = P_t - \frac{(1 - \delta)P_{t+1}}{1 + r}$$

- $r$  is the nominal rate of interest
- $P_{t+1}$  is next year's price for the good

## 2. Asset pricing equilibrium: Housing example

Suppose

- Suppose  $r = .10$  and  $\delta = 0$
- $P_{t+1} = \$ 110 \text{ K}$
- $P_t = \$ 100 \text{ K}$
- What is  $R_t$ ?

## 2. Asset pricing equilibrium: Housing example

Suppose

- Suppose  $r = .10$  and  $\delta = 0$
- $P_{t+1} = \$ 110 \text{ K}$
- $P_t = \$ 100 \text{ K}$
- What is  $R_t$ ?

$$R_t = P_t - \frac{(1 - \delta)P_{t+1}}{1 + r}$$

$$R_t = 100 - \frac{110}{1 + .1}$$

$$R_t = 0$$

## 2. Analyzing Rental Price

We can rearrange the expression to show rental prices depend on three things:

$$R_t = \frac{rP_t + \delta P_{t+1} + P_t - P_{t+1}}{1 + r}$$

- 1 Interest cost<sup>2</sup>:  $rP_t$
- 2 Depreciation:  $\delta P_{t+1}$
- 3 Market re-evaluation:  $P_t - P_{t+1}$

Rental prices are higher, the higher is  $r$ , the greater is the physical rate of depreciation, and the faster the price of the asset is declining



## 2. Analyzing Rental Price: Car example

$$R_t = \frac{rP_t + \delta P_{t+1} + P_t - P_{t+1}}{1 + r}$$

- If cars lose their value quickly (i.e.,  $P_t \gg P_{t+1}$ ), then rental prices will be pretty high

## 2. Analyzing Capital Prices

We can also use the rental price expression to calculate the implied capital price

$$P_t = R_t + \frac{R_{t+1}(1 - \delta)}{(1 + r)} + \frac{R_{t+2}(1 - \delta)^2}{(1 + r)^2} + \dots$$

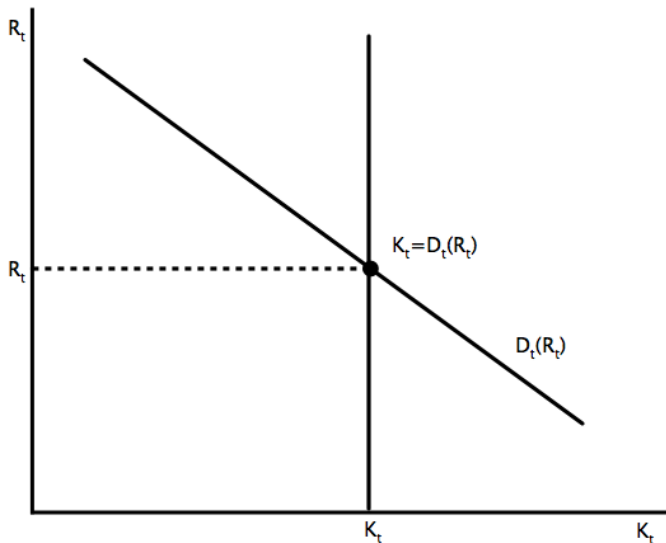
- This equation can be obtained by recursively substituting for future prices in the rental price equation
- This equation should look familiar to you (prices are PV of cash flow stream)
- Capital prices are higher when rental payments to the owner are large and soon

### 3. Rental Market Equilibrium for Housing Services

$$K_t = D(R_t)$$

- The demand for housing services depends on the flow cost of housing services (i.e., the rental rate  $R_t$ ).  $R_t$  is what I pay to use the asset
- Housing services are provided by the stock of housing  $K_t$
- The demand side of the market links the current rental price and the current stock

### 3. Rental Market Equilibrium

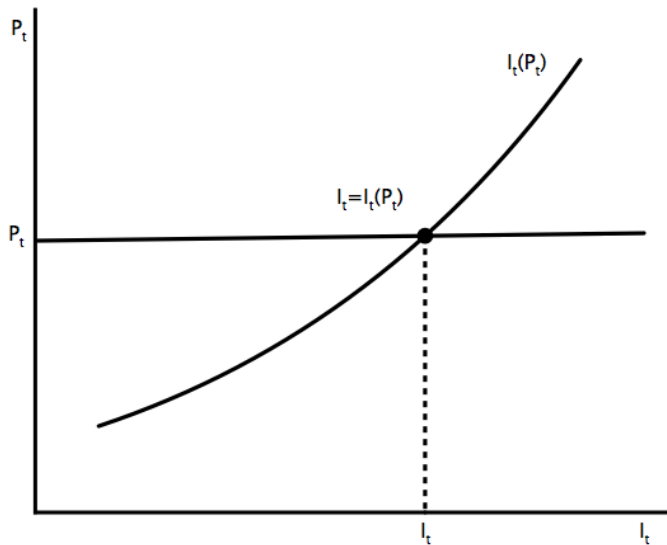


## 4. Investment Market Equilibrium

$$I_t = S(P_t)$$

- The supply of new construction, investment depends on its current price
- Think of this as a new car producer who decides how much to supply based on the current price
- Alternatively, housing construction firms see high house prices and build. They build more when prices are high.

## 4. Investment Market Equilibrium



## 4 key equations

$$K_t = (1 - \delta)K_{t-1} + I_t \quad (1)$$

$$R_t = P_t - \frac{(1 - \delta)P_{t+1}}{1 + r} \quad (2)$$

$$K_t = D(R_t) \quad (3)$$

$$I_t = I(P_t) \quad (4)$$

4 equations and 4 unknowns, but depends on past and the future. Where do past and future come in?

# Market Equilibrium: Past and Future in Housing

- When we look at a market equilibrium for the housing market at any one point in time, we must realize that today's market is influenced by both the past and future
- The effect of the past comes through the effect of past production decisions on the stock of housing
- The effect of the future comes from the effect of future expected rental rates on the current price



# What does the system look like in steady state?

$$\bar{K} = (1 - \delta)\bar{K} + \bar{I}$$

$$\bar{R} = \bar{P} - \frac{(1 - \delta)\bar{P}}{1 + r}$$

$$\bar{K} = D(\bar{R})$$

$$\bar{I} = S(\bar{P})$$

# What does the system look like in steady state?

$$\bar{I} = \delta \bar{K}$$

$$\bar{R} = \bar{P} \left( 1 - \frac{(1 - \delta)}{1 + r} \right)$$

$$\bar{K} = D(\bar{R})$$

$$\bar{I} = S(\bar{P})$$

# What does the system look like in steady state?

We can use the first two equations to plug into the second two equations and obtain the supply and demand in the use market.

$$\begin{aligned}\bar{I} &= \delta \bar{K} \\ \frac{\bar{R}}{\left(1 - \frac{(1-\delta)}{1+r}\right)} &= \bar{P} \\ \bar{K} &= D(\bar{R}) \\ \underbrace{\bar{I}}_{\delta \bar{K}} &= S\left(\underbrace{\bar{P}}_{\frac{\bar{R}}{\left(1 - \frac{(1-\delta)}{1+r}\right)}}\right)\end{aligned}$$

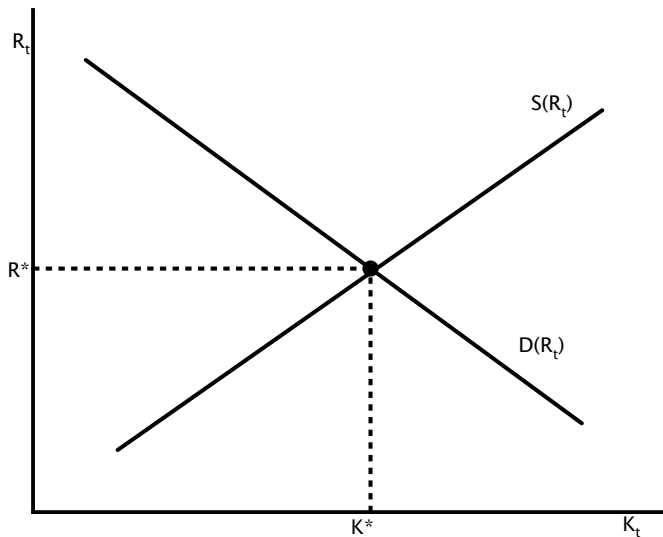
# What does the system look like in steady state?

$$\bar{K} = D(\bar{R})$$

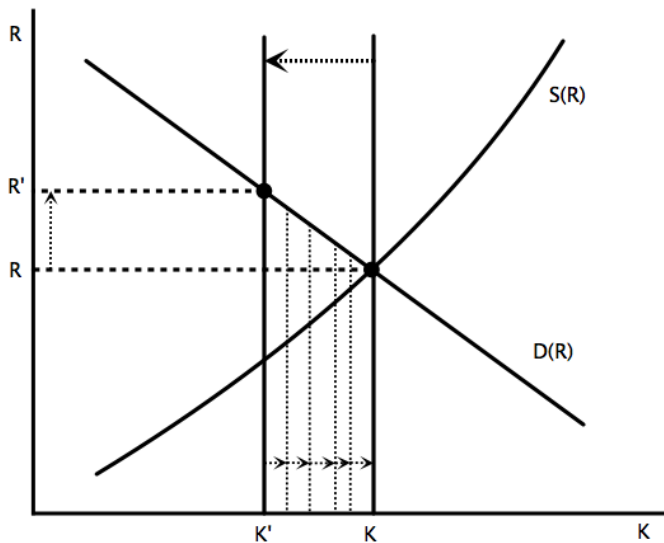
$$\bar{K} = \frac{1}{\delta} S \left( \frac{\bar{R}}{\left(1 - \frac{(1-\delta)}{1+r}\right)} \right)$$

This shows that we have a familiar supply and demand diagram where the quantity is  $K$  and the price is  $R$

# Capital Market Equilibrium



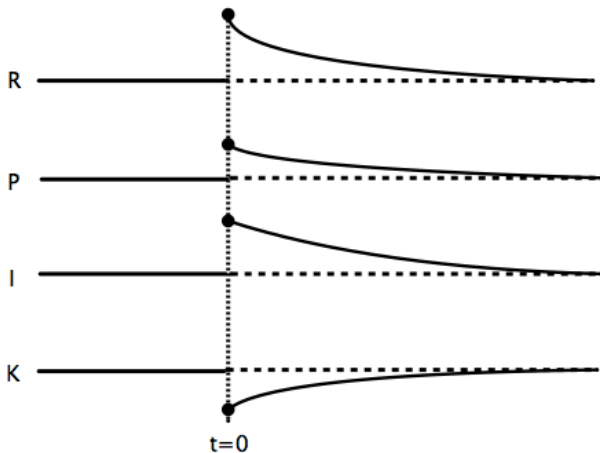
# Earthquake Destroys part of capital stock



# Earthquake Destroys part of capital stock

- The main impact is on the use market. Lower  $K$  increases  $R$ .
- Higher rental prices cause the asset price  $P$  to increase.
- However, since rental rates we decline as we rebuild capital stock, the increase in  $P$  is smaller than increase in  $R$
- Investment follows  $P$ , so it will jump and slowly decline as we rebuild the stock

# Earthquake Destroys part of capital stock





# Speed of Adjustment

What determines the speed of convergence to the steady state?

- 1 **Elasticity of demand** in the rental market  $\varepsilon^D$ . For example, the more the rental price goes up following a destruction of the capital stock, the faster we will converge to steady state (since it will make the capital price go up more, and thereby also investments). With a higher elasticity (in absolute value), the rental price will go up more.
- 2 **Elasticity of supply** in the investment market  $\varepsilon^S$ . This will make investment go up more when the capital price goes up.
- 3 The **depreciation rate**  $\delta$ . This may be the most important aspect, since it puts a lower bound on the speed of convergence. The slowest rate at which the economy ever can return to the steady state is  $\delta$ .

Others examples:

◀ construction costs

◀ interest rates

◀ housing bubble?

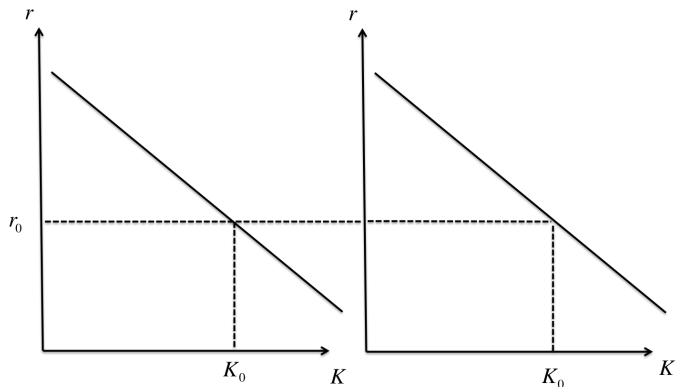
# Simple spatial model: One factor, two locations

# Impact of Capital Tax: One factor, two locations

## Setup

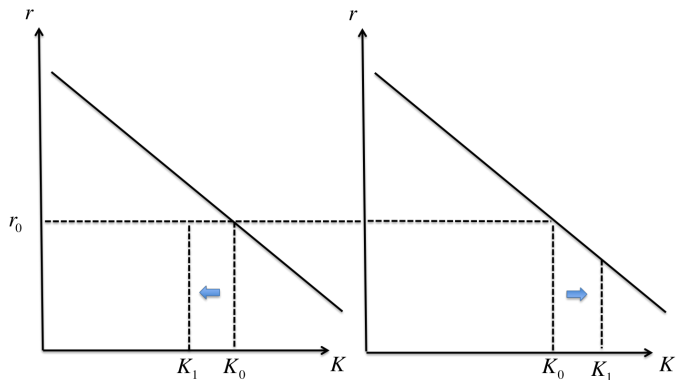
- 1 One factor (capital)
- 2 Two locations: east and west
- 3 Capital market in each location
- 4 Total  $K$  fixed in economy overall

# Initial equilibrium



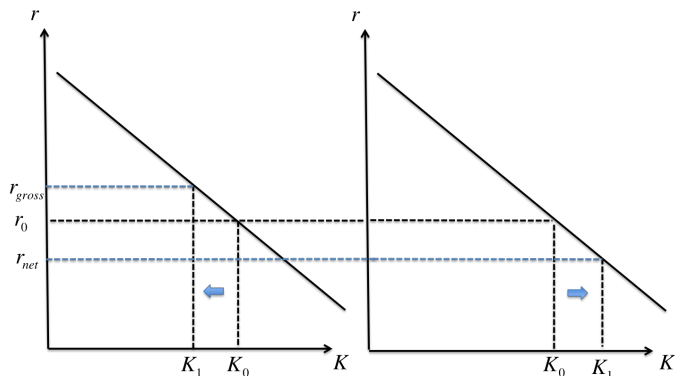
# Tax in west

Causes capital to flee to east



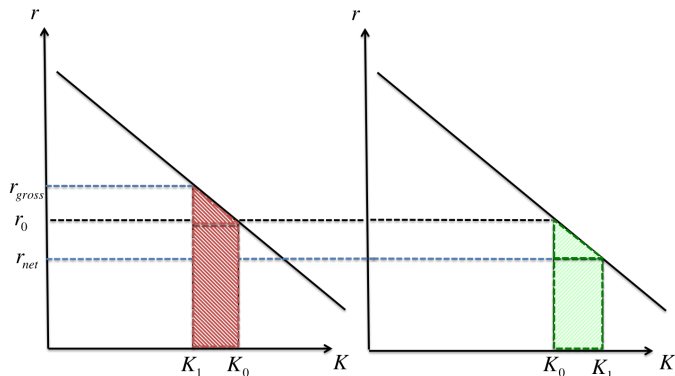
# New allocation of capital

- $K$  flows to east, lowering net returns in both
- Flows continue until after tax return is equalized across markets



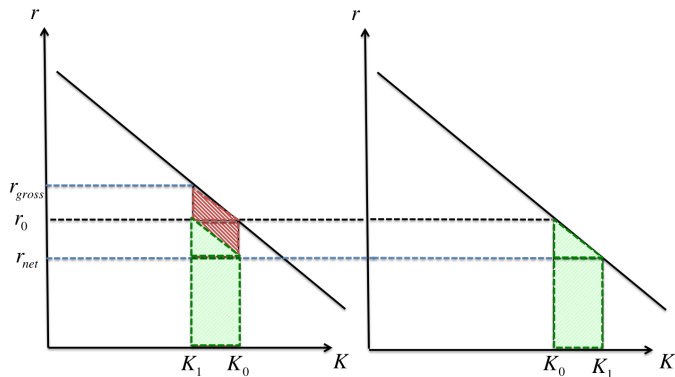
# Welfare changes in each location

- Welfare in west falls by red amount
- Welfare in east increases



# Net welfare changes in aggregate

- Net welfare loss in red





# What determines size of welfare loss in this toy example?

- 1 Size of tax change
- 2 Size of market being taxed (depends on fundamentals)
- 3 Elasticity of demand in both regions (quantity response more generally, which depends on S and D elasticities)
- 4 Strength of complementarities across markets (e.g., labor market)
- 5 Assumptions about effects/value of government spending (assumed to be zero here)
- 6 Presence of existing distortions

Will formalize these ideas in the next section, but this example provides intuition for key forces in the Harberger model

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## 1 Goals

- Characterize effects of corporate tax change in a GE model
- Who bears the burden of corporate taxes? (also capital, output taxes)

## 2 Two sectors (or locations)

- Corporate sector produces output  $X$
- Non-corporate sector produces output  $Y$

## 3 Markets

- Capital: prices  $r_i$ , quantities  $K_i$  where  $i \in \{X, Y\}$
- Labor: prices  $w_i$ , quantities  $L_i$
- Goods: prices  $p_i$ , quantities  $X, Y$

## 4 Agents

- Workers (representative, perfectly mobile, supply 1 unit of labor)
- Firm (representative, perfectly competitive, CRS)

## 5 Equilibrium Conditions

- Good and factor markets clear, factor price equalization
- Consumers max utility, firms earn zero profits

- 1 Harberger is workhorse analytical model: 2 sector and 2 factors
- 2 Fixed supply of capital and labor (short run, closed economy)
- 3 Key intuition is misallocation (magnitude depends on factor intensity, demand elasticities, etc)
- 4 Fullerton and Ta (2017) simplifies Harberger analysis (Cobb Douglas)
- 5 Similar to Hecksher-Ohlin model
- 6 When interpreting as locations not sectors, then implicitly assume no trade costs. Similarly, implicitly assumes no adjustment costs for capital and labor (so long run in that sense)
- 7 Abstracts from amenity or productivity effects of government spending (lump sum rebates or purchases in same share as consumers)

# Fullerton and Ta (2017) Parameterized Harberger Model with Cobb Douglas

# Consumers: Preferences and Budget Constraint

Utility of representative worker is  $U = X^\gamma Y^{1-\gamma}$

- $X$  is corporate sector output
- $Y$  is non-corporate sector output

Budget constraint is  $p_x X + p_y Y = I$

- $I$  is income, which is sum of labor and capital income
- $p_i$  is price of output in sector  $i$  where  $i \in \{X, Y\}$

Workers have fixed expenditure shares (e.g.  $I\gamma$ ); demand for  $X$  and  $Y$  is:

$$X = \frac{I\gamma}{p_x}$$
$$Y = \frac{I(1-\gamma)}{p_y}$$

N.B. note no labor supply or saving decision

- Indirect utility is given

$$V(p_x, p_y, I) = \left(\frac{I\gamma}{p_x}\right)^\gamma \left(\frac{I(1-\gamma)}{p_y}\right)^{1-\gamma} = \frac{I}{\bar{p}}$$

where  $\bar{p} = \left(\frac{p_x}{\gamma}\right)^\gamma \left(\frac{p_y}{1-\gamma}\right)^{1-\gamma}$  is the “ideal” price index

- Inverting indirect utility (i.e.,  $V = \frac{I}{\bar{p}}$ ), gives the expenditure function  $I = E$ :

$$E(\bar{p}, U) = U\bar{p}$$

- So  $\bar{p}$  is the price paid for each “util”

# Firms maximize profits

- Corporate sector solves:

$$\max_{K_x, L_x} (1 - \tau_X)p_x X - (1 + \tau_K + \tau_{KX})rK_x - wL_x, \text{ where } X = AK_x^\alpha L_x^{1-\alpha}$$

- where

$\tau_X$  = tax on output of  $X$

$\tau_K$  = tax on capital

$\tau_{KX}$  = tax on capital in production of  $X$

- Non-corporate sector solves:

$$\max_{K_y, L_y} p_y Y - (1 + \tau_K)rK_y - wL_y, \text{ where } Y = BK_y^\beta L_y^{1-\beta}$$



# Firm optimization (and factor demand)

FOCs:

$$w = (1 - \tau_X)p_x(1 - \alpha)A \left( \frac{K_x}{L_x} \right)^\alpha$$

$$w = p_y(1 - \beta)B \left( \frac{K_y}{L_y} \right)^\beta$$

and

$$(1 + \tau_K + \tau_{KX})r = (1 - \tau_X)p_x\alpha A \left( \frac{L_x}{K_x} \right)^{1-\alpha}$$

$$(1 + \tau_K)r = p_y\beta B \left( \frac{L_y}{K_y} \right)^{1-\beta}$$

# Exogenous parameters

- Taxes:  $\tau_X, \tau_K, \tau_{KX}$ 
  - $\tau_X$  is tax on corporate sector output (sales tax)
  - $\tau_K$  is tax on capital
  - $\tau_{KX}$  is tax on capital used in corporate sector
- Consumer Parameter:  $\gamma$ 
  - $\gamma$  governs importance of corporate goods for utility
  - $1 - \gamma$  governs importance of non-corporate goods for utility
- Firm Parameters:  $\alpha, \beta, A, B$ 
  - $\alpha$  is output elasticity of capital in sector  $X$
  - $1 - \alpha$  output elasticity of labor in sector  $X$
  - $\beta$  output elasticity of capital in sector  $Y$
  - $1 - \beta$  output elasticity of labor in sector  $Y$
  - $A$  and  $B$  are productivity in corp and non-corp sectors
- Endowments;  $K, L$ 
  - $K$  is total capital
  - $L$  is total labor

# Endogenous Model Outcomes and Equilibrium

Endogenous outcomes are  $K_i, L_i, p_i, X, Y, w, r$ :

- Capital: prices  $r_i$ , quantities  $K_i$  where  $i \in \{X, Y\}$
- Labor: prices  $w_i$ , quantities  $L_i$
- Goods: prices  $p_i$ , quantities  $X, Y$

*Given  $\tau_X, \tau_K, \tau_{KX}, \gamma, \alpha, \beta, A, B, K, L$ , equilibrium is defined by prices and quantities  $\{w, r, p_i, K_x, K_y, L_x, L_y, X, Y\}$  such that good and factor markets clear and firms and workers optimize.*

# Equilibrium: closed form expressions

# Ten Equations and ten unknowns (with taxes)

In log terms, the equations are:

$$\ln X = \ln I + \ln \gamma - \ln p_x$$

$$\ln Y = \ln I + \ln(1 - \gamma) - \ln p_y$$

$$\ln K_x - \ln L_x - \ln w + \ln r = \ln \alpha - \ln(1 - \alpha) - \ln(1 + \tau_K + \tau_{KX})$$

$$\ln K_y - \ln L_y - \ln w + \ln r = \ln \beta - \ln(1 - \beta) - \ln(1 + \tau_K)$$

$$\ln X - \ln K_x + \ln p_x - \ln r = \ln(1 + \tau_K + \tau_{KX}) - \ln \alpha - \ln(1 - \tau_X)$$

$$-\ln X + \ln L_x - \ln p_x + \ln w = \ln(1 - \alpha) + \ln(1 - \tau_X)$$

$$\ln Y - \ln K_y + \ln p_y - \ln r = \ln(1 + \tau_K) - \ln \beta$$

$$-\ln Y + \ln K_y - \ln p_y + \ln w = \ln(1 - \beta)$$

where  $K = K_x + K_y$  and  $L = L_x + L_y$ .

◀ Equations without taxes

## Solutions with taxation (1/2)

- Given taxes  $\tau_K$ ,  $\tau_X$ , and  $\tau_{KX}$ , we have

$$X = A\gamma(1 - \tau_X) \left( \frac{\alpha(1 + \tau_K)}{\alpha\gamma(1 - \tau_X)(1 + \tau_K) + \beta(1 - \gamma)(1 + \tau_K + \tau_{KX})} K \right)^\alpha \left( \frac{(1 - \alpha)}{(1 - \alpha)\gamma(1 - \tau_X) + (1 - \beta)(1 - \gamma)} L \right)^{1 - \alpha}$$

$$Y = B(1 - \gamma) \left( \frac{\beta(1 + \tau_K + \tau_{KX})}{\alpha\gamma(1 - \tau_X)(1 + \tau_K) + \beta(1 - \gamma)(1 + \tau_K + \tau_{KX})} K \right)^\beta \left( \frac{(1 - \beta)}{(1 - \alpha)\gamma(1 - \tau_X) + (1 - \beta)(1 - \gamma)} L \right)^{1 - \beta}$$

$$K_x = \frac{\alpha\gamma(1 - \tau_X)(1 + \tau_K)}{\alpha\gamma(1 - \tau_X)(1 + \tau_K) + \beta(1 - \gamma)(1 + \tau_K + \tau_{KX})} K$$

$$K_y = \frac{\beta(1 - \gamma)(1 + \tau_K + \tau_{KX})}{\alpha\gamma(1 - \tau_X)(1 + \tau_K) + \beta(1 - \gamma)(1 + \tau_K + \tau_{KX})} K$$

$$L_x = \frac{(1 - \alpha)\gamma(1 - \tau_X)}{(1 - \alpha)\gamma(1 - \tau_X) + (1 - \beta)(1 - \gamma)} L$$

## Solutions with taxation (1/2)

$$L_y = \frac{(1 - \beta)(1 - \gamma)}{(1 - \alpha)\gamma(1 - \tau_X) + (1 - \beta)(1 - \gamma)} L$$
$$p_x = \frac{I}{A(1 - \tau_X)} \left( \frac{\alpha\gamma(1 - \tau_X)(1 + \tau_K) + \beta(1 - \gamma)(1 + \tau_K + \tau_{KX})}{\alpha(1 + \tau_K)K} \right)^\alpha$$
$$\left( \frac{(1 - \alpha)\gamma(1 - \tau_X) + (1 - \beta)(1 - \gamma)}{(1 - \alpha)L} \right)^{1 - \alpha}$$
$$p_y = \frac{I}{B} \left( \frac{\alpha\gamma(1 - \tau_X)(1 + \tau_K) + \beta(1 - \gamma)(1 + \tau_K + \tau_{KX})}{\beta(1 + \tau_K + \tau_{KX})K} \right)^\beta$$
$$\left( \frac{(1 - \alpha)\gamma(1 - \tau_X) + (1 - \beta)(1 - \gamma)}{(1 - \beta)L} \right)^{1 - \beta}$$
$$w = \frac{I}{L} [(1 - \alpha)\gamma(1 - \tau_X) + (1 - \beta)(1 - \gamma)]$$
$$r = \frac{I}{K} \left[ \frac{\alpha\gamma(1 - \tau_X)(1 + \tau_K) + \beta(1 - \gamma)(1 + \tau_K + \tau_{KX})}{(1 + \tau_K)(1 + \tau_K + \tau_{KX})} \right]$$

# Welfare Loss from Taxation



# Equivalent variation and Burden of taxation

- Equivalent variation  $EV$  is the change in wealth at initial prices that would be equivalent to the price change in terms of utility.

$$\begin{aligned}EV &= E(\bar{p}^0, U^1) - \bar{I} \\ &= \bar{p}^0 U^1 - \bar{p}^0 U^0 = \bar{p}^0 (U^1 - U^0)\end{aligned}$$

where  $\bar{p}^0$  and  $\bar{p}^1$  are the “ideal” prices in period 0 and 1

- Use  $-EV$  as a positive measure of tax burden, so

$$EB = -EV = \bar{p}^0 (U^0 - U^1)$$

Amount that burden exceeds tax revenues is called excess burden (Auerbach and Hines, 2002). We consider a revenue-neutral reform with a distorting tax where all revenue is returned lump sum, so net revenue is zero and thus, net loss is excess burden.

# Average and Marginal Excess Burden

- Average Excess Burden (*AEB*) is the total welfare loss from the tax divided by the total revenue collected by the government:

$$AEB = \frac{EB}{R}$$

where  $\bar{p}^0$  and  $\bar{p}^1$  are the “ideal” prices in period 0 and 1

- Marginal excess burden (*MEB*) measures the effects of a small change in the tax rate on burden:

$$\Delta EB = \bar{p}^0 (EB^1 - EB^2)$$
$$MEB = \frac{\Delta EB}{\Delta R}$$

N.B. See Hendren’s recent TPE paper for more detailed discussion

# Understanding Equilibrium: Graphical and quantitative analysis

# Understanding Equilibrium: Graphs and numerical example

- There are a lot of moving parts
- Helpful to think about relative factor markets (relative prices and relative quantities) in the two sectors
- Will start with demand side, then supply side, then analyze equilibrium graphically pre and post taxes
- Will work with a calibrated version of the model to do quantitative analysis

Taking ratios of each sector's FOCs gives:

$$\frac{w}{r} = \frac{(1 - \alpha)}{\alpha} \left( \frac{L_x}{K_x} \right)^{-1} (1 + \tau_K + \tau_{KX}) \quad (5)$$

$$\frac{w}{r} = \frac{(1 - \beta)}{\beta} \left( \frac{L_y}{K_y} \right)^{-1} (1 + \tau_K) \quad (6)$$

Recall

$$\begin{aligned}L &= L_x + L_y \\ K &= K_x + K_y\end{aligned}$$

Thus, the economy-wide labor capital ratio is:

$$\begin{aligned}\frac{L}{K} &= \frac{L_x}{K} + \frac{L_y}{K} \\ \frac{L}{K} &= \frac{L_x}{K_x} \left( \frac{K_x}{K} \right) + \frac{L_y}{K_y} \left( \frac{K_y}{K} \right)\end{aligned}\tag{7}$$

This says that overall labor to capital ratio is a weighted average of the labor to capital ratio in both sectors

# Factor Market Prices and Quantities

We can invert 5 and 6 to get  $L_x/K_x$  and  $L_y/K_y$  as functions of  $w/r$ . Then

$$\frac{L}{K} = \left(\frac{w}{r}\right)^{-1} \left( \frac{(1 + \tau_k + \tau_{kx})(1 - \alpha)}{\alpha} \frac{K_x}{K} + \frac{(1 + \tau_k)(1 - \beta)}{\beta} \frac{K_y}{K} \right) \quad (8)$$

In equilibrium, we found

$$\frac{K_x}{K} = \frac{\alpha\gamma(1 - \tau_X)(1 + \tau_K)}{\alpha\gamma(1 - \tau_X)(1 + \tau_K) + \beta(1 - \gamma)(1 + \tau_K + \tau_{KX})} \quad (9)$$

$$\frac{K_y}{K} = \frac{\beta(1 - \gamma)(1 + \tau_K + \tau_{KX})}{\alpha\gamma(1 - \tau_X)(1 + \tau_K) + \beta(1 - \gamma)(1 + \tau_K + \tau_{KX})} \quad (10)$$

# Numerical Example

- $X$  and  $Y$  produced given functions

$$X = AK_x^{.6} L_x^{.4}$$

$$Y = BK_y^{.2} L_y^{.8}$$

- Identical households have utility:

$$U = X^{.5} Y^{.5}$$

- Fixed<sup>3</sup> level of income  $I = 2,400$
- Demand for  $X$  and  $Y$  is given by:

$$X = \frac{2400(.5)}{p_x} = \frac{1200}{p_x}$$

$$Y = \frac{2400(.5)}{p_y} = \frac{1200}{p_y}$$

- Assume unity of prices in the initial state ( $p_x = p_y = r = w = 1$ )



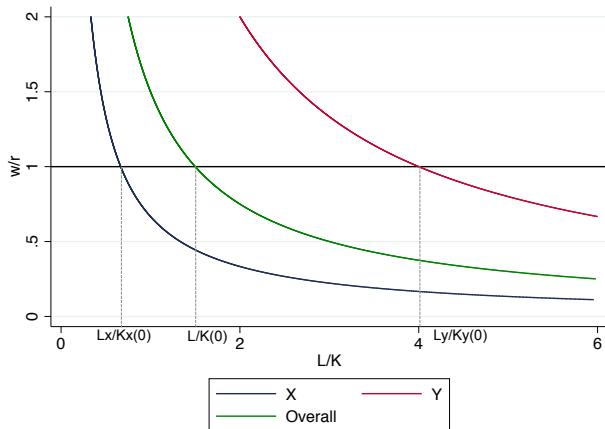
## Initial quantities and prices ( $\tau_K = \tau_{KX} = \tau_X = 0$ )

- With this parameterization of utility and technology, we have:

	Value
$L_x$	480
$L_y$	960
$K_x$	720
$K_y$	240
$X$	1200
$Y$	1200
$p_x$	1
$p_y$	1
$r$	1
$w$	1

- Use the values above to derive  $A \approx 1.96$  and  $B \approx 1.69$  (given output, inputs, and production function)

# Initial Factor Market Equilibrium ( $\tau_K = \tau_{KX} = \tau_X = 0$ )



**Figure:** Wage to Rent Ratio in both sectors and economy overall

$$\frac{L_x}{K_x} = \frac{2}{3}, \frac{K_x}{K} = \frac{3}{4}, \frac{L_y}{K_y} = 4, \text{ and } \frac{K_y}{K} = \frac{1}{4}, \text{ so } \frac{L}{K} = \frac{2}{3} \times \frac{3}{4} + 4 \times \frac{1}{4} = 1.5.$$

## Effect of Tax on Corporate Output ( $\tau_X = .3$ )

- 1  $\tau_X$  reduces demand for  $X$
- 2 We will have factors move from producing  $X$  to producing  $Y$  until prices and quantities re-equilibrate
- 3 Specifically, since  $w_x = w_y \Rightarrow (1 + \tau_x)p_x MPL_x = p_y MPL_y$ , we need a combination of lower  $p_x$  and higher  $MPL_x$  (and thus lower factor demand in  $x$ ) and/or higher  $p_y$  and lower  $MPL_y$
- 4 The movement of both factors to  $Y$  increases the weight of the non-corporate sector in labor and capital demand (see eq 8 and dashed green line in next slide)
- 5 Since the non-corporate sector is relatively labor intensive, total relative labor demand increases
- 6 Hence, the equilibrium wage to rental ratio increases

# Tax on Corporate Output

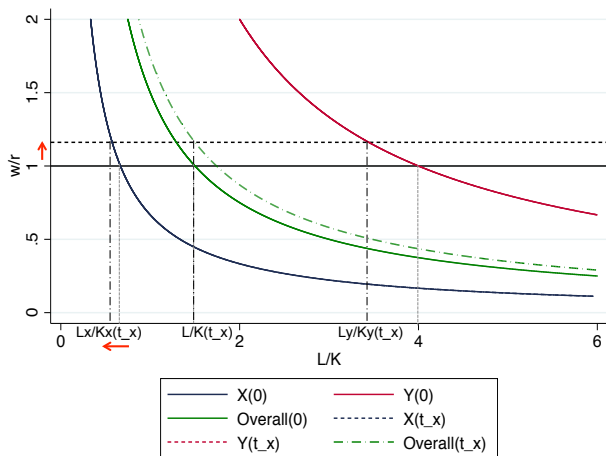


Figure: Relative Factor market equilibrium with  $\tau_X = .3, \tau_K = \tau_{KX} = 0$

# Tax on Corporate Output: Prices and Quantities

<i>Panel A: Allocations and Prices</i>			
	$t_x = 0$	$t_x = .3$	$t_x = .31$
$L_x$	480	373.333	369.368
$L_y$	960	1,066.667	1,070.632
$K_x$	720	650.323	647.296
$K_y$	240	309.677	312.704
$X$	1,200	1,020.942	1,013.750
$Y$	1,200	1,373.811	1,380.577
$p_x$	1	1.175	1.184
$p_y$	1	0.874	0.869
$r$	1	0.775	0.768
$w$	1	0.900	0.897
$w/r$	1	1.161	1.168
$L_x/K_x$	0.667	0.574	0.571
$L_y/K_y$	4	3.444	3.424
$L/K$	1.500	1.500	1.500

# Tax on Corporate Output: Welfare and Burden Measures

<i>Panel B: Exact Measures of Welfare</i>			
	$t_X = 0$	$t_X = .3$	$t_X = .31$
$\bar{p}^0$	2		
$\bar{p}$	2	2.026	2.029
$U$	1,200	1,184.306	1,183.030
$EB$	0	31.387	33.940
$R$	0	360	372
$AEB$		0.087	0.091
$MEB$			0.213

# Effect of Tax on Capital

- 1 Suppose a tax on all capital:  $\tau_K = .3$ , and  $\tau_{KX} = \tau_X = 0$
- 2 Both sectors face tax on capital, so capital allocation across sectors does not change (see 9 and 10 in which the  $(1 + \tau_k)$  terms cancel)
- 3  $(1 + \tau_k)$  increases relative labor demand symmetrically in eq 8 in both sectors (i.e., it shifts up  $L_i/K_i$ ), so factor allocation stays constant and all adjustment is through relative prices
- 4 In this case, capital fully bears the burden of the tax (i.e.,  $w/r$  rises by 30% to offset tax increase)

N.B. remember that in these examples, the *overall* stock of capital is fixed. In practice, investment and firm creation respond to taxes. A key question is how much they respond

# Tax on Capital

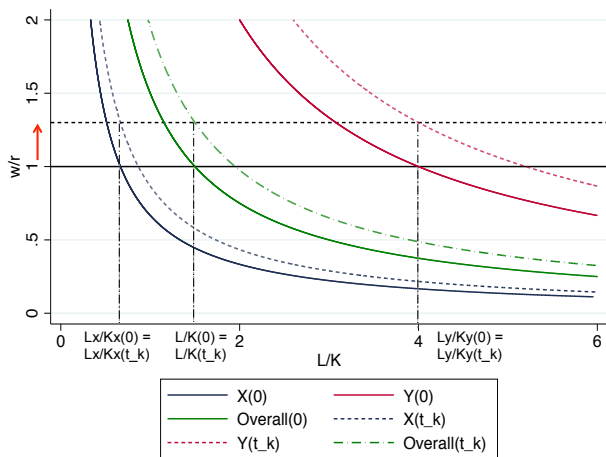


Figure: Relative Factor market equilibrium with  $\tau_K = .3, \tau_{KX} = \tau_X = 0$



# Tax on Capital: Allocation and Quantities

<i>Panel A: Allocations and Prices</i>			
	$t_K=0$	$t_K=.3$	$t_K=.31$
$L_x$	480	480	480
$L_y$	960	960	960
$K_x$	720	720	720
$K_y$	240	240	240
$X$	1,200	1,200	1,200
$Y$	1,200	1,200	1,200
$p_x$	1	1	1
$p_y$	1	1	1
$r$	1	0.769	0.763
$w$	1	1	1
$w/r$	1	1.300	1.310
$L_x/K_x$	0.667	0.667	0.667
$L_y/K_y$	4	4	4
$L/K$	1.500	1.500	1.500

# Tax on Capital: Welfare and Burden Measures

<i>Panel B: Exact Measures of Welfare</i>			
	$t_K=0$	$t_K=.3$	$t_K=.31$
$\bar{p}^0$	2		
$\bar{p}$	2	2	2
$U$	1,200	1,200	1,200
$EB$	0	0	0
$R$	0	221.538	227.176
$AEB$		0	0
$MEB$			0

# Effect of Tax on Corporate Capital

- 1 Now suppose a tax on corporate capital,  $\tau_{KX} = .3$
- 2 Corporate sector demands less capital ( $r_x = \frac{p_x MPK_x}{1 + \tau_{KX}}$ ), so capital flows from corporate to non-corporate sector (see eq 9 and 10)
- 3 Lower capital allocation to producing  $X$  increases the weight of the non-corporate sector in labor and capital demand (see eq 8 and dashed green line in next slide)
- 4 Causes misallocation (too much  $K_y$  and thus, too much  $Y$ , not enough  $X$ ), which reduces welfare as in prior example

# Tax on Corporate Capital

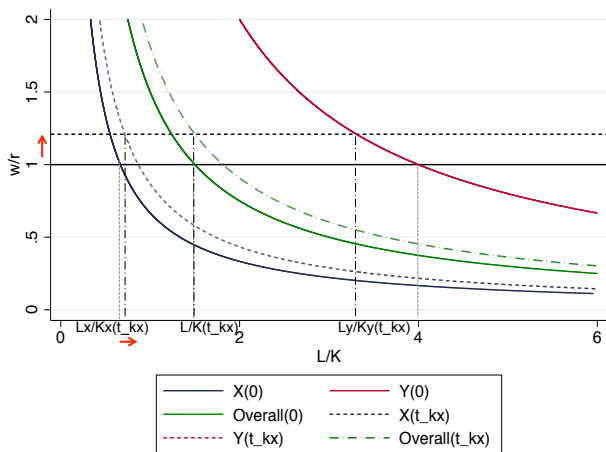


Figure: Relative Factor market equilibrium with  $\tau_{KX} = .3, \tau_K = \tau_X = 0$

# Tax on Corporate Capital: Allocation and Quantities

<i>Panel A: Allocations and Prices</i>			
	$t_{KX} = 0$	$t_{KX} = .3$	$t_{KX} = .31$
$L_x$	480	480	480
$L_y$	960	960	960
$K_x$	720	669.767	668.214
$K_y$	240	290.233	291.786
$X$	1,200	1,149.043	1,147.442
$Y$	1,200	1,246.488	1,247.820
$p_x$	1	1.044	1.046
$p_y$	1	0.963	0.962
$r$	1	0.827	0.823
$w$	1	1	0.897
$w/r$	1	1.209	1.090
$L_x/K_x$	0.667	0.717	0.718
$L_y/K_y$	4	3.308	3.290
$L/K$	1.500	1.500	1.500

# Tax on Corporate Capital: Welfare and Burden Measures

<i>Panel B: Exact Measures of Welfare</i>			
	$t_{KX} = 0$	$t_{KX} = .3$	$t_{KX} = .31$
$\bar{p}^0$	2		
$\bar{p}$	2	2.005	2.006
$U$	1,200	1,174.245	1,173.427
$EB$	0	51.510	53.146
$R$	0	166.154	170.382
$AEB$		0.310	0.312
$MEB$			0.387

# Harberger Model

# Harberger (more general utility and technology)

Ten equations needed for equilibrium are:

$$p_x X + p_y Y = wL + rK \quad (11)$$

$$MRS_{XY} = \frac{p_x(1 + \tau_X)}{p_y} \quad (12)$$

$$c_x(w, r(1 + \tau_K + \tau_{KX})) = p_x \quad (13)$$

$$c_y(w, r(1 + \tau_K)) = p_y \quad (14)$$

$$w = p_x F_{xL} \quad (15)$$

$$w = p_y F_{yL} \quad (16)$$

$$r(1 + \tau_K + \tau_{KX}) = p_x(1 - \tau_X) F_{xK} \quad (17)$$

$$r = p_y F_{yK} \quad (18)$$

$$K = K_x + K_y \quad (19)$$

$$L = L_x + L_y \quad (20)$$



# Definitions

- Share of income spent on  $X$  and  $Y$ :

$$s_x \equiv \frac{p_x X}{p_x X + p_y Y}, \quad s_y \equiv \frac{p_y Y}{p_x X + p_y Y}, \quad s_x + s_y = 1$$

- Share of income from labor and capital:

$$s_w \equiv \frac{wL}{wL + rK}, \quad s_r \equiv \frac{rK}{wL + rK}$$

- Cost shares in production of  $X$  and  $Y$ :

$$\theta_L \equiv \frac{wL_x}{wL_x + rK_x}, \quad \theta_K \equiv \frac{rK_x}{wL_x + rK_x}, \quad \theta_x + \theta_y = 1$$
$$\phi_L \equiv \frac{wL_y}{wL_y + rK_y}, \quad \phi_K \equiv \frac{rK_y}{wL_y + rK_y}, \quad \phi_x + \phi_y = 1$$

- Share of labor and capital used to produce  $X$ :

$$\lambda_L \equiv \frac{L_x}{L}, \quad \lambda_K \equiv \frac{K_x}{K}$$

- By Euler's Theorem and CRS, we also have:

$$p_x X = wL_x + rK_x, \quad p_y Y = wL_y + rK_y$$

$$\Rightarrow \lambda_L = \frac{s_x \theta_L}{s_x \theta_L + s_y \phi_L} = \frac{s_x (1 - \theta_K)}{1 - s_x \theta_K - s_y \phi_K}$$

$$\Rightarrow \lambda_K = \frac{s_x \theta_K}{s_x \theta_K + s_y \phi_K} = \frac{s_x (1 - \theta_L)}{1 - s_x \theta_L - s_y \phi_L}$$

$$s_x(\hat{p}_x + \hat{X}) + s_y(\hat{p}_y + \hat{Y}) = s_w \hat{w} + s_r \hat{R}$$

$$\hat{X} - \hat{Y} = \sigma_D(\hat{p}_y - \hat{p}_x - d\tau_X)$$

$$\hat{p}_x = \theta_L \hat{w} + \theta_K(\hat{r} + d\tau_K + d\tau_{KX})$$

$$\hat{p}_y = \phi_L \hat{w} + \phi_K(\hat{r} + d\tau_K)$$

$$\lambda_L \hat{L}_x + (1 - \lambda_L) \hat{L}_y = 0$$

$$\lambda_K \hat{K}_x + (1 - \lambda_K) \hat{K}_y = 0$$

$$\hat{L}_x = \hat{X} + \theta_K \sigma_X(\hat{r} + d\tau_K + d\tau_{KX} - \hat{w})$$

$$\hat{K}_x = \hat{X} + \theta_L \sigma_X(\hat{w} - d\tau_K - d\tau_{KX} - \hat{r})$$

$$\hat{L}_y = \hat{Y} + \phi_K \sigma_Y(\hat{r} + d\tau_K - \hat{w})$$

$$\hat{K}_y = \hat{Y} + \phi_L \sigma_Y(\hat{w} - d\tau_K - \hat{r})$$

# Matrix Form of the System of Linear Equations

$$\begin{bmatrix}
 \sigma_D & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 \\
 1 & -\theta_L & -\theta_K & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & -\phi_L & -\phi_K & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & \lambda_L & 0 & 1-\lambda_L & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & \lambda_K & 0 & 1-\lambda_K \\
 0 & \theta_K \sigma_X & -\theta_K \sigma_X & -1 & 0 & 1 & 0 & 0 & 0 \\
 0 & -\theta_L \sigma_X & \theta_L \sigma_X & -1 & 0 & 0 & 1 & 0 & 0 \\
 0 & \phi_K \sigma_Y & -\phi_K \sigma_Y & 0 & -1 & 0 & 0 & 1 & 0 \\
 0 & -\phi_L \sigma_Y & \phi_L \sigma_Y & 0 & -1 & 0 & 0 & 0 & 1
 \end{bmatrix}
 \begin{bmatrix}
 \hat{p}_x \\
 \hat{w} \\
 \hat{f} \\
 \hat{X} \\
 \hat{Y} \\
 \hat{L}_x \\
 \hat{K}_x \\
 \hat{L}_y \\
 \hat{K}_y
 \end{bmatrix}
 =
 \begin{bmatrix}
 -\sigma_X \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0
 \end{bmatrix}
 d\tau_X +
 \begin{bmatrix}
 0 \\
 -\theta_K \\
 0 \\
 0 \\
 0 \\
 \theta_K \sigma_X \\
 -\theta_L \sigma_X \\
 0 \\
 0
 \end{bmatrix}
 d\tau_{KX}$$

where we eliminated equation 1 by Walras law and normalized  $p_y = 1$ , so  $\hat{p}_y = 0$

# Two Main Effects of Taxing $K_x$

- 1 **Substitution effects:** capital bears incidence
- 2 **Output effects:** capital may not bear all incidence

# Substitution effects

- Tax on  $K_x$  shifts production in  $X$  away from  $K$  so aggregate demand for  $K$  goes down
- Because total  $K$  is fixed,  $r$  falls  $\rightarrow K$  bears some of the burden

Another intuition for this is that capital is misallocated across sectors, which lowers  $r$  and  $rK$

- Tax on  $K_x$  makes  $X$  more expensive
- Demand shifts to  $Y$
- *Case 1:*  $K_x/L_x > K_y/L_y$  ( $X$ : cars,  $Y$ : bikes)
  - $X$  more capital intensive  $\rightarrow$  lower aggregate demand for  $K$
  - Output + subst. effect:  $K$  bears the burden of the tax
- *Case 2:*  $K_x/L_x < K_y/L_y$  ( $X$ : bikes,  $Y$ : cars)
  - $X$  less capital intensive  $\rightarrow$  higher aggregate demand for  $K$
  - Subst. and output effects have opposite signs  $\rightarrow$  labor may bear some the tax

*Harberger showed that under a variety of reasonable assumptions, capital bears exactly 100 percent of the tax. Note that this is the burden on all capital – as capital flees the corporate sector, it depresses returns in the noncorporate sector as well. Both the realism of the model and the characterization of the corporate income tax as an extra tax on capital in the corporate sector are subject to question, as discussed in considerable detail by the subsequent literature on the effects of the corporate tax. – Alan Auerbach*

See Auerbach TPE paper on who bears the corporate tax for more details



# Appendix:

# Ten equations and ten unknowns (without taxes)

In log terms, the equations are:

$$\begin{aligned}\ln X &= \ln A + \alpha \ln K_x + (1 - \alpha) \ln L_x \\ &= \ln I + \ln \gamma - \ln p_x\end{aligned}$$

$$\begin{aligned}\ln Y &= \ln B + \beta \ln K_y + (1 - \beta) \ln L_y \\ &= \ln I + \ln(1 - \gamma) - \ln p_y\end{aligned}$$

$$\begin{aligned}\ln w &= \ln p_x + \ln(1 - \alpha) + \ln A + \alpha(\ln K_x - \ln L_x) \\ &= \ln p_y + \ln(1 - \beta) + \ln B + \beta(\ln K_y - \ln L_y)\end{aligned}$$

$$\begin{aligned}\ln r &= \ln p_x + \ln \alpha + \ln A + (1 - \alpha)(\ln L_x - \ln K_x) \\ &= \ln p_y + \ln \beta + \ln B + (1 - \beta)(\ln L_y - \ln K_y)\end{aligned}$$

where  $K = K_x + K_y$  and  $L = L_x + L_y$ .

## Solutions in initial equilibrium without taxes (1/2)

- Solving for the system of equations, quantities are:

$$X = A\gamma \left( \frac{\alpha K}{\alpha\gamma + \beta(1-\gamma)} \right)^\alpha \left( \frac{(1-\alpha)L}{(1-\alpha)\gamma + (1-\beta)(1-\gamma)} \right)^{1-\alpha}$$

$$Y = B(1-\gamma) \left( \frac{\beta K}{\alpha\gamma + \beta(1-\gamma)} \right)^\beta \left( \frac{(1-\beta)L}{(1-\alpha)\gamma + (1-\beta)(1-\gamma)} \right)^{1-\beta}$$

$$K_x = \frac{\alpha\gamma}{\alpha\gamma + \beta(1-\gamma)} K$$

$$K_y = \frac{\beta(1-\gamma)}{\alpha\gamma + \beta(1-\gamma)} K$$

$$L_x = \frac{(1-\alpha)\gamma}{(1-\alpha)\gamma + (1-\beta)(1-\gamma)} L$$

$$L_y = \frac{(1-\beta)(1-\gamma)}{(1-\alpha)\gamma + (1-\beta)(1-\gamma)} L$$

## Solutions in initial equilibrium without taxes (2/2)

Prices are:

$$p_x = \frac{I}{A} \left( \frac{\alpha\gamma + \beta(1-\gamma)}{\alpha K} \right)^\alpha \left( \frac{(1-\alpha)\gamma + (1-\beta)(1-\gamma)}{(1-\alpha)L} \right)^{1-\alpha}$$

$$p_y = \frac{I}{B} \left( \frac{\alpha\gamma + \beta(1-\gamma)}{\beta K} \right)^\beta \left( \frac{(1-\alpha)\gamma + (1-\beta)(1-\gamma)}{(1-\beta)L} \right)^{1-\beta}$$

$$w = \frac{I}{L} ((1-\alpha)\gamma + (1-\beta)(1-\gamma))$$

$$r = \frac{I}{K} (\alpha\gamma + \beta(1-\gamma))$$

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# Increase the cost of new construction

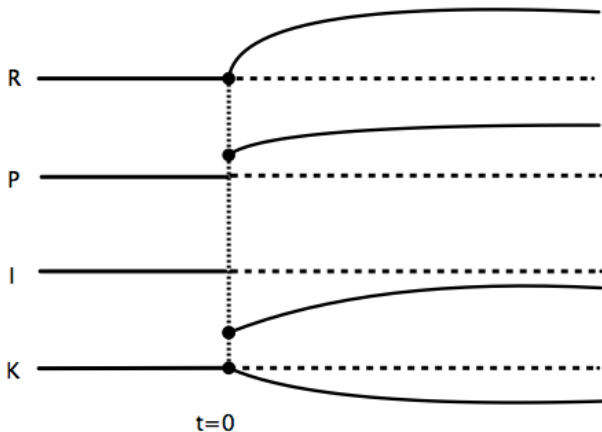
Suppose the economy is in steady state and suddenly the costs of new construction increase.

# Increase the cost of new construction

Suppose the economy is in steady state and suddenly the costs of new construction increase.

- The main impact is on the asset market. Supply curve shifts left.
- Lower supply decreases new construction
- The lower stock of capital causes rents to rise
- Rising rents and the prospects of future higher rents (remember long-run rents are higher) cause the price to rise
- As prices rise over time, construction rebounds

# Increase the cost of new construction



## Decline in the interest rate

Suppose the economy is in steady state and suddenly the interest rate  $r$  declines.



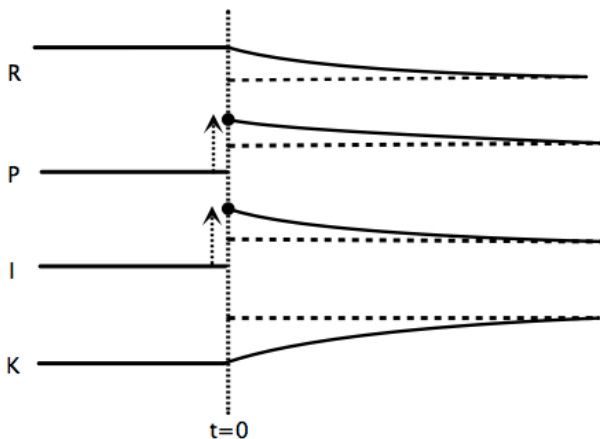
# Decline in the interest rate

Suppose the economy is in steady state and suddenly the interest rate  $r$  declines.

- Lower  $r \Rightarrow$  future streams of payments are more valuable, so  $P$  will jump up
- The jump in asset prices causes investment to jump up
- More investment increases the capital stock
- Higher capital stocks start to decrease rental rates
- Higher rental rates decrease asset prices
- We will end at a steady state with higher  $K^*$  and lower  $R^*$

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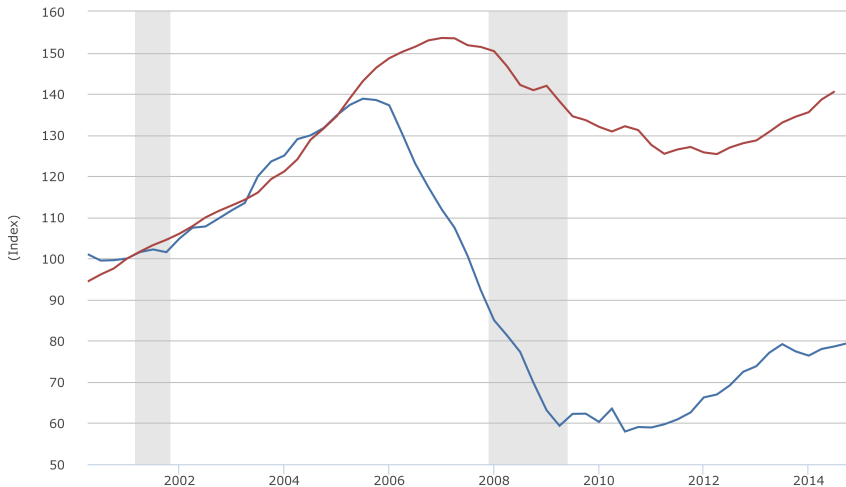
# Decline in the interest rate



# US House Prices and Residential Investment

FRED 

— Real Private Residential Fixed Investment, 2001:Q1=100  
— All-Transactions House Price Index for the United States, 2001:Q1=100



Shaded areas indicate US recessions - 2015 [research.stlouisfed.org](http://research.stlouisfed.org)

**Was this an irrational bubble?**

What did we see in the **housing boom** in the 2000s?

What did we see in the **housing boom** in the 2000s?

- Low interest rates and high capital prices
- Therefore, housing services are cheap to use and investment will increase as  $S(P)$  increases with  $P$
- Suppose people expected higher future demand
- More downward pressure on rental prices, higher capital prices, more construction

# US House Prices and Residential Investment

What did we see in the **housing bust** in the late 2000s?

# US House Prices and Residential Investment

What did we see in the **housing bust** in the late 2000s?

- We've build up a large housing stock
- Suppose now the anticipated increase in demand never comes
- $\Rightarrow$  Falling house prices
- $\Rightarrow$  Big decline in investment
- $\Rightarrow$  Also makes consuming housing services more expensive
- $\Rightarrow$  Things will adjust as housing stock goes back to the steady state

**Was this an irrational bubble?**

Observing a crash in the housing market does not tell us whether (1) there were rational expectations about future demand (coupled with low interest rates) or (2) a bubble (that could not be justified by expectations). [◀ Back](#)