Lecture 1: The Armington Model

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1 Introduction

The Armington model (Armington, 1969) is based on the premise that each country produces a different good and consumers would like to consume at least some of each country's goods. This assumption is of course ad hoc, and it completely ignores the "classical" trade forces such as increased specialization due to comparative advantage. However, as we will see, the model (when combined with Constant Elasticity of Substitution (CES) preferences as in (Anderson, 1979))[1] provides a nice characterization of trade flows between many countries.

Before jumping to the model, it is helpful to provide a brief motivation of why we are interested in writing down a flexible model in the first place. "Classical" trade theories (Ricardo, Heckscher-Ohlin), while extremely useful in highlighting the economic forces behind trade, are very difficult to generalize to a set-up with many trading partners and bilateral trade costs. Because the real world clearly has both of these, the classical theories do not provide much guidance in doing empirical work. Because of this difficulty, those doing empirical work in trade began using a statistical (i.e. atheoretic) model known as the **gravity equation** due to its similarity the Newton's law of gravitation. The gravity equation states that total trade flows from country i to country j, X_{ij}, are proportional to the product of the origin country's GDP Y_i and destination country's GDP Y_j and inversely proportional[2] to the distance between the two countries, D_{ij}:

$$X_{ij} = \alpha \frac{Y_i \times Y_j}{D_{ij}}. \tag{1}$$

For a variety of reasons (which we will go into later on in the course), this gravity equation is often estimated in a more general form, which I will refer to as the **generalized gravity**

[1] Actually, in the main text, Anderson (1979) considers Cobb-Douglass preferences and writes that "there is little point in the exercise" of generalizing to CES preferences, doing so only in an appendix. Despite his reluctance to do so, the paper has been cited thousands of times as the example of an Armington model with CES preferences.

[2] This is actually in contrast to Newton's law of gravitation, where the force of gravity is inversely proportional to the *square* of the distance.

equation:

$$X_{ij} = K_{ij}\gamma_i\delta_j, \tag{2}$$

where K_{ij} is a measure of the resistance of trade between i and j, γ_i is an origin fixed effect and δ_j is a destination fixed effect.

The gravity equation (1) and its generalization (2) have proven to be enormously successful at explaining a large fraction of the variation in observed bilateral trade flows; indeed, it is probably not too much of an exaggeration to say that the gravity equation is one of the most successful empirical relationships in all of economics. Because it was originally proposed as a statistical relationship, however, the absence of a theory justifying the relationship made it very difficult to ask any meaningful counterfactual questions; e.g. "what would happen to trade between i and j if the tariff was lowered between i and k?"

This is why the Armington model (as formulated by (Anderson, 1979)) was so important: it provided the first theoretical foundation for the gravity relationship. It is also a great place to start our course, as one of the great surprises of the international trade literature over the past fifteen years has been how robust the results first present in the Armington model are across different quantitative trade models.

A final word before I begin presenting the model. This course is organized so that we first consider the various microeconomic foundations of the gravity equation (Part I) and then consider the general equilibrium properties of the equilibrium (Part II). Hence, for the next few classes, we will be deriving gravity equations (which will depend on equilibrium variables); we won't actually close the model until Part II.

2 Model Set-up

Let us now turn to the set-up of the model.

2.1 The world

Suppose there is a compact set S of countries. For now, I will assume that S is discrete, although having a continuum of countries does not change much. As much as possible, I will refer to an origin country as i and a destination country as j and order the subindices such that X_{ij} is the trade from i to j. For whatever reason, however, the naming conventions and order conventions are not universal, so beware of this when reading papers.

2.2 Supply

The **Armington assumption** is that each country $i \in S$ produces a distinct variety of a good. Because countries map one-to-one to varieties, I will index the varieties by their country names (this will not be true later on when we have to keep track both of varieties and countries).

Suppose that each country $i \in S$ is populated by a measure of L_i workers, which we assume is exogenous. Throughout the course, we will assume that each worker supplies her unit of labor inelastically. Furthermore, let the productivity of a worker (i.e. how much of a good each worker can produce) be A_i, which we also assume is exogenous. Let the wage of a worker in country i be w_i. The wage will be determined in equilibrium.

For now, we assume that labor is the only factor of production (we will add intermediate inputs later on). There are three common assumptions made about the market structure. The first is that markets in every country are perfectly competitive, so the price of a good is simply equal to its marginal cost. The second is that each country is endowed with a certain quantity of a good and chooses the amount to sell to each destination to maximize profits. The third is that production is monopolistically competitive. We will consider each below.

Finally, suppose that there are **iceberg trade costs** $\{\tau_{ij}\}_{i,j\in S}$. This means that in order from one unit of a good to arrive in destination j, destination i must ship τ_{ij} units. Iceberg trade costs are so called because a fraction $\tau_{ij} - 1$ "melts" on its way from i to j, much as if you were towing an iceberg. We almost always assume that $\tau_{ij} \geq 1$ and usually assume that $\tau_{ii} = 1$ for all $i \in S$, i.e. trade with oneself is costless. Furthermore, we sometimes assume that the following triangle inequality holds: for all $i, j, k \in S$: $\tau_{ij}\tau_{jk} \geq \tau_{ik}$. The triangle inequality says that it is never cheaper to ship a good via an intermediate location rather than sell directly to a destination.

2.3 Demand

Throughout most of the course, we will assume that workers have Constant Elasticity of Substitution (CES) preferences. Why do we do so? As will become evident as you work on the first problem set, CES preferences have a number of attractive properties: (1) they are homothetic; (2) they nest a number of special demand systems (e.g. Cobb-Douglass); and (3) they are extremely tractable. However, it is important to note up front that I do not think any trade economist actually believes preferences are CES; we just use them out of convenience.

In particular, assume that each country has a representative consumer who gets utility U_j from the consumption of goods shipped from all countries $i \in S$:

$$U_j = \left(\sum_{i \in S} a_{ij}^{\frac{1}{\sigma}} q_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{3}$$

where $\sigma \in (0, \infty)$ is the elasticity of substitution and a_{ij} is an exogenous preference shifter. A couple of things to note: first, q_{ij} is the *quantity* of a good shipped from i that *arrives* in j (the amount shipped is $\tau_{ij} q_{ij}$); second, the fact that there is a representative consumer is not particularly important: we can always assume that workers (with identical preferences) are the ones consuming the goods. In this case, the per capita welfare U_j^{pc} becomes:

$$U_j^{pc} = \left(\sum_{i \in S} a_{ij}^{\frac{1}{\sigma}} \left(\frac{q_{ij}}{L_j}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \iff$$
$$U_j^{pc} L_j = \left(\sum_{i \in S} a_{ij}^{\frac{1}{\sigma}} \left(q_{ij}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \iff$$
$$U_j = \left(\sum_{i \in S} a_{ij}^{\frac{1}{\sigma}} \left(q_{ij}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

so that U_j can be interpreted as the total welfare of country j (note the importance of homotheticity in this derivation!).

3 Gravity

To get a generalized gravity equation that takes the form of equation (2) requires two steps: first, we need to solve the representative consumer's utility maximization problem, which will tell us how much a consumer demands of each good as a function of its price. Second, we solve for the optimal price given the market structure.

3.1 Optimal demand

We now solve the representative consumer's utility maximization problem. Given the importance of CES in the class, I think it useful to do the full derivation. Let the income of country j be denoted Y_j and let the price of a good (net of trade costs) from country i in country j be p_{ij}. Then the utility maximization problem is:

$$\max_{\{X_{ij}\}_{i \in S}} \left(\sum_{i \in S} a_{ij}^{\frac{1}{\sigma}} q_{ij}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \quad \text{s.t.} \sum_{i \in S} q_{ij} p_{ij} \leq Y_j,$$

where I ignore the constraint that $X_{ij} > 0$ (why is this okay?).

The Lagrangian is:

$$\mathcal{L} : \left(\sum_{i \in S} a_{ij}^{\frac{1}{\sigma}} q_{ij}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} - \lambda \left(\sum_{i \in S} q_{ij} p_{ij} - Y_j\right)$$

First order conditions (FOCs) are:

$$\frac{\partial \mathcal{L}}{\partial q_{ij}} = 0 \iff \left(\sum_{i \in S} a_{ij}^{\frac{1}{\sigma}} q_{ij}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} a_{ij}^{\frac{1}{\sigma}} q_{ij} = \lambda p_{ij}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \iff Y_j = \sum_{i \in S} q_{ij} p_{ij}$$

From the first FOC we have for any $i, i' \in S$:

$$\frac{a_{ij}^{\frac{1}{\sigma}} q_{ij}^{-\frac{1}{\sigma}}}{a_{i'j}^{\frac{1}{\sigma}} q_{i'j}^{-\frac{1}{\sigma}}} = \frac{p_{ij}}{p_{i'j}} \iff$$
$$\frac{a_{ij}}{a_{i'j}} = \frac{p_{ij}^{\sigma}}{p_{i'j}^{\sigma}} \frac{q_{ij}}{q_{i'j}}$$

Rearranging and multiplying both sides by $p_{i'j}$ yields:

$$q_{i'j} p_{i'j} = \frac{1}{a_{ij}} q_{ij} p_{ij}^{\sigma} a_{i'j} p_{i'j}^{1-\sigma}$$

Summing over all $i' \in S$ yields:

$$\sum_{i' \in S} q_{i'j} p_{i'j} = \frac{1}{a_{ij}} q_{ij} p_{ij}^{\sigma} \sum_{i' \in S} a_{i'j} p_{i'j}^{1-\sigma} \iff$$
$$Y_j = \frac{1}{a_{ij}} q_{ij} p_{ij}^{\sigma} P_j^{1-\sigma}$$

where the last line used the second FOC and $P_j \equiv \left(\sum_{i' \in S} a_{i'j} p_{i'j}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ is known as the Dixit-Stiglitz price index. You will show in your problem set that $U_j = \frac{Y_j}{P_j}$, i.e. dividing income by the price index gives the total welfare of country j. Rearranging the last line yields the **CES demand function**:

$$q_{ij} = a_{ij} p_{ij}^{-\sigma} Y_j P_j^{\sigma-1}, \tag{4}$$

Equation (4) implies that the quantity consumed in j of a good produced in i will be increasing with $j's$ preference for the good (a_{ij}), decreasing with the price of the good (p_{ij}), increasing with $j's$ income (Y_j), and increasing with $j's$ price index. [Class question: why is the quantity demanded increasing in the price index?].

Noting that the *value* of total trade is simply equal to the price times quantity, i.e. $X_{ij} = p_{ij} q_{ij}$, equation (4) yields:

$$X_{ij} = a_{ij} p_{ij}^{1-\sigma} Y_j P_j^{\sigma-1}. \tag{5}$$

Note that equation (5) is almost to the point of being a (generalized) gravity equation; the only thing left to do is to solve for p_{ij}.

3.2 Optimal supply

We now determine the equilibrium prices for three different market structures: (1) perfect competition; (2) an endowment economy; and (3) monopolistic competition. We will see that all three yield very similar equilibrium prices.

3.2.1 Perfect competition

Suppose that the market for each country/good is perfectly competitive, so that the price of a good is simply the marginal cost. Because each worker can produce A_i units and costs w_i in terms of a wage, the marginal cost of production is simply $\frac{w_i}{A_i}$. This implies that the price at the factory door (i.e. without shipping costs) is $p_i = \frac{w_i}{A_i}$. What about with trade costs? Recall that with the iceberg formulation, $\tau_{ij} \geq 1$ units have to be shipped in order for one unit to arrive. This means that $\tau_{ij} \geq 1$ units have to be produced in country i in order for one unit to be consumed in country j. Hence the price in country j of consuming one unit from country i is:

$$p_{ij} = \tau_{ij} \frac{w_i}{A_i}. \tag{6}$$

Note that this implies that:

$$\frac{p_{ij}}{p_i} = \tau_{ij}, \tag{7}$$

i.e. the ratio of the price in any destination relative to the price at the factory door is simply equal to the iceberg trade cost. Equation (7) is called a **no-arbitrage equation**, as it means that there is no way for an individual to profit by buying a good in country i and sell in country j (or vice versa). Note, however, that there may still be profitable trading opportunities between triplets of countries even if equation (7) holds when the triangle inequality is not satisfied.

Substituting equation (6) into equation (5) yields our first generalized gravity equation of the course:

$$X_{ij} = a_{ij} \tau_{ij}^{1-\sigma} \left(\frac{w_i}{A_i} \right)^{1-\sigma} Y_j P_j^{\sigma-1}. \tag{8}$$

To the extent that trade costs are increasing in distance, the value of bilateral trade flows will decline as long as $\sigma > 1$. The greater the value of σ, the more the decline in trade flows [Class question: what is the intuition for this?].

3.2.2 Endowment

Suppose now that each country only has a fixed amount, while I will call Q_i and chooses how to allocate it across consumers in all countries in order to maximize profits (which are equal to revenue, as there are no costs) taking the demand in each country as given. That is, country i chooses how much to export to each country by solving the following maximization problem:

$$\max_{\{q_{ij}\}_{j \in S}} \sum_{j \in S} p_{ij} q_{ij} \text{ s.t. } \sum_j \tau_{ij} q_{ij} \leq Q_i \text{ and } q_{ij} = a_{ij} p_{ij}^{-\sigma} Y_j P_j^{\sigma-1}.$$

Note that the country needs to account for the fact that some of its endowment will melt away due to the iceberg trade costs. Substituting the second constraint into the maximand and first constraint yields the equivalent problem of what price to set in each country:

$$\max_{\{p_{ij}\}_{j \in S}} \sum_{j \in S} a_{ij} p_{ij}^{1-\sigma} Y_j P_j^{\sigma-1} \text{ s.t. } \sum_j \tau_{ij} a_{ij} p_{ij}^{-\sigma} Y_j P_j^{\sigma-1} \leq Q_i$$

First order conditions with respect to p_{ij} yield:

$$\begin{aligned}(1-\sigma)\, a_{ij} p_{ij}^{-\sigma} Y_j P_j^{\sigma-1} &= -\lambda \tau_{ij} \sigma p_{ij}^{-\sigma-1} Y_j P_j^{\sigma-1} \iff \\ p_{ij} &= \frac{\sigma}{\sigma-1} \lambda \tau_{ij}, \end{aligned} \tag{9}$$

i.e. the price net of trade costs in all destinations is equal (which immediately implies the no-arbitrage equation (7) above). Substituting equation (9) into the endowment constraint yields:

$$\begin{aligned}\sum_j \left(\frac{\sigma}{\sigma-1}\lambda\tau_{ij}\right)^{-\sigma} \tau_{ij} a_{ij} Y_j P_j^{\sigma-1} &= Q_i \iff \\ \left(\frac{\sigma}{\sigma-1}\lambda\right) &= \left(\frac{\sum_j \tau_{ij}^{1-\sigma} a_{ij} Y_j P_j^{\sigma-1}}{Q_i}\right)^{\frac{1}{\sigma}} \iff \\ p_{ij} &= \left(\frac{\sum_j \tau_{ij}^{1-\sigma} a_{ij} Y_j P_j^{\sigma-1}}{Q_i}\right)^{\frac{1}{\sigma}} \tau_{ij} \end{aligned} \tag{10}$$

Substituting equation (10) into equation (5) yields our second generalized gravity equation of the course:

$$X_{ij} = a_{ij} \tau_{ij}^{1-\sigma} \left(\sum_j \tau_{ij}^{1-\sigma} a_{ij} Y_j P_j^{\sigma-1}\right)^{\frac{1-\sigma}{\sigma}} Q_i^{\frac{\sigma-1}{\sigma}} Y_j P_j^{\sigma-1}. \tag{11}$$

Note that this gravity equation closely resembles the one for perfect competition; the only difference is that the origin wage has been replaced with an object that depends on the demand in all other regions. (As we will see, the equilibrium origin wage also depends on demand in all other regions, so the two equations are actually even more similar than they appear).

We can actually use equation (11) to get even closer to the (non-general) gravity equation. Suppose that $\tau_{ii} = 1$. Because the price net of transportation costs is the same across all destinations, the origin income is simply equal to the product of its endowment and the price in i, i.e.:

$$\begin{aligned} Y_i &= Q_i p_{ii} \iff \\ Y_i &= Q_i^{\frac{\sigma-1}{\sigma}} \left(\sum_j \tau_{ij}^{1-\sigma} a_{ij} Y_j P_j^{\sigma-1}\right)^{\frac{1}{\sigma}}, \end{aligned} \tag{12}$$

where the second line used the equilibrium price from equation (10). Substituting equation (12) into the gravity equation (11) yields:

$$X_{ij} = a_{ij} \tau_{ij}^{1-\sigma} \times \frac{Y_i}{\Pi_i^{1-\sigma}} \times \frac{Y_j}{P_j^{1-\sigma}}, \tag{13}$$

where $\Pi_i \equiv \left(\sum_j \tau_{ij}^{1-\sigma} a_{ij} Y_j P_j^{\sigma-1}\right)^{\frac{1}{1-\sigma}}$ bears a striking resemblance to the price index... [insert foreshadowing here]. Equation (13) is actually about as close as we will ever get to the original gravity equation. This is because all of our theories say that bilateral trade flows depend on more than just the bilateral trade costs and the incomes of the exporter and importer; what also matters is so-called "bilateral resistance": intuitively, the greater the cost of exporting in general, the smaller the $\Pi_i^{1-\sigma}$; conversely, the greater the cost of importing in general, the smaller the $P_i^{1-\sigma}$. This means that trade between any two countries depends not only on the incomes of those two countries but also the "cost" of trading between those countries *relative to* trading with all other countries. This point was made in the enormously famous and influential paper "Gravity with Gravitas: A Solution of the Border Puzzle" (Anderson and Van Wincoop, 2003).

3.2.3 Monopolistic competition

Finally, suppose that each country $i \in S$ produces its differentiated variety at a constant marginal cost c_i (which you can think of as the $\frac{w_i}{A_i}$ if you would like, but it could be more general) and chooses how much to sell to all destinations in order to maximize its profits, taking as given the CES consumer demand (4) in each location. This is known as monopolistic competition.

The optimization problem faced by country i is:

$$\max_{\{q_{ij}\}_{j\in S}} \sum_{j\in S} \left(p_{ij}q_{ij} - c_i\tau_{ij}q_{ij}\right) \text{ s.t. } q_{ij} = a_{ij}p_{ij}^{-\sigma}Y_jP_j^{\sigma-1}$$

Note that the total marginal cost of i producing a good for consumption in j is $c_i\tau_{ij}$. As with the endowment economy, we can substitute the constraint into the maximand and write the equivalent unconstrained problem of choosing the price to sell to each location as:

$$\max_{\{p_{ij}\}_{j\in S}} \sum_{j\in S} \left(a_{ij}p_{ij}^{1-\sigma}Y_jP_j^{\sigma-1} - c_i\tau_{ij}a_{ij}p_{ij}^{-\sigma}Y_jP_j^{\sigma-1}\right)$$

Note that the constant marginal cost assumption implies that the country can treat each destination as a separate optimization problem (this will come in helpful in models we will see later on). The first order conditions are:

$$\begin{aligned}(1-\sigma)\,a_{ij}p_{ij}^{-\sigma}Y_jP_j^{\sigma-1} &= -\sigma c_i\tau_{ij}a_{ij}p_{ij}^{-\sigma-1}Y_jP_j^{\sigma-1} \iff \\ p_{ij} &= \frac{\sigma}{\sigma-1}c_i\tau_{ij}. \end{aligned} \tag{14}$$

Hence, with monopolistic competition, the country charges a price that is a constant markup $\frac{\sigma}{\sigma-1}$ above marginal cost. [Class question: Note that the markup falls as the elasticity of substitution increases; what is the intuition?] As with the endowment economy and perfect competition, the optimal price satisfies the no-arbitrage equation (7).

Substituting the price equation (14) into the gravity equation implied by CES demand (5) yields another gravity equation:

$$X_{ij} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} a_{ij}\tau_{ij}^{1-\sigma}c_i^{1-\sigma}Y_jP_j^{\sigma-1} \tag{15}$$

If we interpret $c_i = \frac{w_i}{A_i}$ then gravity equation (15) is almost identical to gravity equation (8) with perfect competition; the only difference here is that all trade flows are smaller (if $\sigma > 1$) as a result of the markups.

4 Next steps

The Armington model, while based on the ad-hoc assumption that consumers intrinsically want to consume goods from all countries, provides a theoretical foundation for the empirical gravity relationship. In addition, because many of the components present in the Armington model (e.g. CES demand) play an important role in the (more realistic) models that follow, the Armington model presents a great introduction to modern international trade. In the next class, we will dispense with the Armington assumption and introduce firms into the model following the work of Krugman (1980).

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References

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